Hanoi University of Science and Technology **School of Transportation Engineering** **Exam No. 1 No. of pages: 1**		**FINAL EXAM** **TE4240: Vehicle dynamics** *Date: 14.06.2018* *Duration: 90 min*
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Vehicle dynamics answers

3 differential equations describe the movements of sprung mass (**3 points**):

$$m_a\ddot{z} = F_{C1} + F_{K1} + F_{C2} + F_{K2} + F_{C3} + F_{K3} + F_{C4} + F_{K4} \quad (1)$$

$$I_y\ddot{\varphi}_y = \left(F_{C3} + F_{K3} + F_{C4} + F_{K4}\right)b - \left(F_{C1} + F_{K1} + F_{C2} + F_{K2}\right)a \quad (2)$$

$$I_x\ddot{\varphi}_x = \left(F_{C2} + F_{K2} + F_{C3} + F_{K3}\right)\frac{w}{2} - \left(F_{C1} + F_{K1} + F_{C4} + F_{K4}\right)\frac{w}{2} \quad (3)$$

2 differential equations describe the movements of front sprung masses (**2 points**):

$$m_{11}\ddot{\xi}_{11} = \left(F_{CL1} + F_{KL1}\right) - \left(F_{C1} + F_{K1}\right) \quad (4)$$

$$m_{12}\ddot{\xi}_{12} = \left(F_{CL2} + F_{KL2}\right) - \left(F_{C2} + F_{K2}\right) \quad (5)$$

2 differential equations describe the movements of rear sprung masses (**2 points**):

$$m_2\ddot{\xi}_2 = \left(F_{CL3} + F_{KL3} + F_{CL4} + F_{KL4}\right) - \left(F_{C3} + F_{K3} + F_{C4} + F_{K4}\right) \quad (6)$$

$$I_{x2}\ddot{\varphi}_2 = \left(F_{CL3} + F_{KL3} - F_{C3} - F_{K3}\right)\frac{w}{2} - \left(F_{CL4} + F_{KL41} - F_{C4} - F_{K4}\right)\frac{w}{2} \quad (7)$$

Suspension forces (**1 points**)

$$F_{C1} = \left(\xi_{11} - \left(z - a\varphi_y - \frac{w}{2}\varphi_x\right)\right)C_1; \qquad F_{K1} = \left(\dot{\xi}_{11} - \left(\dot{z} - a\dot{\varphi}_y - \frac{w}{2}\dot{\varphi}_x\right)\right)K_1;$$

$$F_{C2} = \left(\xi_{12} - \left(z - a\varphi_y + \frac{w}{2}\varphi_x\right)\right)C_2; \qquad F_{K2} = \left(\dot{\xi}_{12} - \left(\dot{z} - a\dot{\varphi}_y + \frac{w}{2}\dot{\varphi}_x\right)\right)K_2;$$

$$F_{C3} = \left(\left(\xi_2 + \frac{w}{2}\dot{\varphi}_2\right) - \left(z + b\varphi_y + \frac{w}{2}\varphi_x\right)\right)C_3; \quad F_{K3} = \left(\left(\dot{\xi}_2 + \frac{w}{2}\dot{\varphi}_2\right) - \left(\dot{z} + b\dot{\varphi}_y + \frac{w}{2}\dot{\varphi}_x\right)\right)K_3;$$

$$F_{C4} = \left(\left(\xi_2 - \frac{w}{2}\dot{\varphi}_2\right) - \left(z + b\varphi_y - \frac{w}{2}\varphi_x\right)\right)C_4; \quad F_{K4} = \left(\left(\dot{\xi}_2 - \frac{w}{2}\dot{\varphi}_2\right) - \left(\dot{z} + b\dot{\varphi}_y - \frac{w}{2}\dot{\varphi}_x\right)\right)K_4;$$

Vertical force, in the case of the separation of wheels from the road surface (**2 points**)

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$$F_{CL1}+F_{KL1}=\begin{cases} C_{L1}(h_1-\xi_{11})+K_{L1}(\dot{h}_1-\dot{\xi}_{11}) & khi \quad h_1-\left(\xi_{11}-g(\dfrac{m_{11}+m_a/4}{C_{L1}})\right)\geq 0 \\ 0 & khi \quad h_1-\left(\xi_{11}-g(\dfrac{m_{11}+m_a/4}{C_{L1}})\right)<0 \end{cases}$$

$$F_{CL2}+F_{KL2}=\begin{cases} C_{L2}(h_2-\xi_{12})+K_{L2}(\dot{h}_2-\dot{\xi}_{12}) & khi \quad h_2-\left(\xi_{12}-g(\dfrac{m_{12}+m_a/4}{C_{L2}})\right)\geq 0 \\ 0 & khi \quad h_2-\left(\xi_{12}-g(\dfrac{m_{12}+m_a/4}{C_{L2}})\right)<0 \end{cases}$$

$$F_{CL3}+F_{KL3}=\begin{cases} C_{L3}(h_3-\xi_2-\dfrac{\mathrm{w}}{2}\varphi_2)+K_{L3}(\dot{h}_3-\dot{\xi}_2-\dfrac{\mathrm{w}}{2}\dot{\varphi}_2) & khi \quad h_3-\left(\xi_2+\dfrac{\mathrm{w}}{2}\varphi_2-g(\dfrac{m_2/2+m_a/4}{C_{L3}})\right)\geq 0 \\ 0 & khi \quad h_3-\left(\xi_2+\dfrac{\mathrm{w}}{2}\varphi_2-g(\dfrac{m_2/2+m_a/4}{C_{L3}})\right)<0 \end{cases}$$

$$F_{CL4}+F_{KL4}=\begin{cases} C_{L4}(h_4-\xi_2+\dfrac{\mathrm{w}}{2}\varphi_2)+K_{L4}(\dot{h}_4-\dot{\xi}_2+\dfrac{\mathrm{w}}{2}\dot{\varphi}_2) & khi \quad h_4-\left(\xi_2-\dfrac{\mathrm{w}}{2}\varphi_2-g(\dfrac{m_2/2+m_a/4}{C_{L3}})\right)\geq 0 \\ 0 & khi \quad h_4-\left(\xi_2-\dfrac{\mathrm{w}}{2}\varphi_2-g(\dfrac{m_2/2+m_a/4}{C_{L3}})\right)<0 \end{cases}$$

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