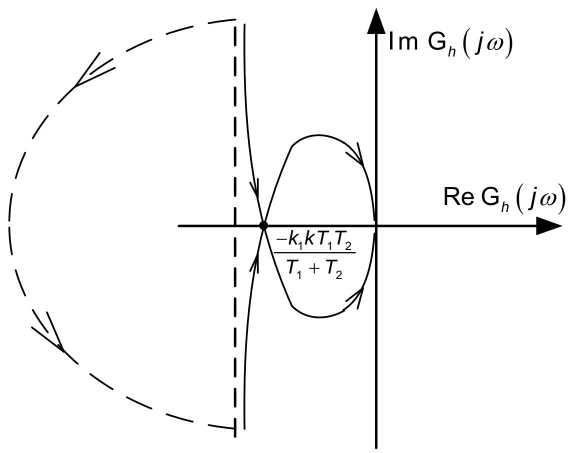
SOLUTION OF EXAM 01
Subject: CONTROL THEORY

Exercise 1 **(5 marks)**:

a) (1.5 marks)

i. **(1.5 đ)**

i. **(1đ)** We have:



(0.5đ) Using Nyquist Property $0 < k_1 < 2$

i. (0.5 đ)

Because of stable closed loop system, there exists the Limitation

$$\lim_{t\to\infty}\left(u(t)-y(t)\right) = \lim_{s\to 0} s\left(U(s)-Y(s)\right) = \lim_{s\to 0} sU(s)\left(1-\frac{k_2 k_1 G(s)}{1+k_1 G(s)}\right);$$

$u(t) = 1(t)$ và $\lim_{s\to 0} G(s) = +\infty$, we obtain $\lim_{t\to\infty}\left(u(t)-y(t)\right) = 1-k_2$

We imply $k_2 = 1$;

b) (2 marks)

i. **(1 mark)** $G(s) = \dfrac{k}{s(1+T_2 s)^2}$

$a = 4: \quad T_I = T_1 + 4T_2,\ k_p = \dfrac{T_I}{8kT_2^2},\ T_D = \dfrac{4T_1T_2}{T_I},\ T = 4T_2$ with

$k = 0.5,\ T_1 = T_2 = 2$ we obtain $T_I = 10,\ k_p = \dfrac{5}{8},\ T_D = 1.6,\ T = 8$

ii. **(1 mark)** Stability reserve of closed system does not depend on $R_2(s)$. We need to obtain the open system with transfer function:

$$G_h(s) = R_1(s)G(s) = k_p\left(1+\frac{1}{T_I s}+T_D s\right)\cdot\frac{k}{s(1+T_1 s)(1+T_2 s)}$$

$$= \frac{k_p(1+T_A s)(1+T_B s)}{T_I s}\cdot\frac{k}{s(1+T_1 s)(1+T_2 s)} = \frac{k_p k(1+T_B s)}{T_I s^2(1+T_2 s)}$$

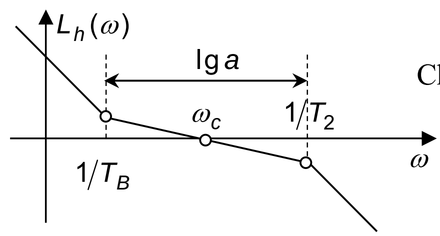
We select $T_A = T_1$ with $T_A + T_B = T_I$, $T_A T_B = T_I T_D$, $T_B = 4T_2 > T_2$, we obtain the Bode diagram of open system as follows.

$$\omega_c = \frac{1}{\sqrt{T_B T_2}}.$$

$T_B = 8,\ T_1 = T_2 = 2$ we imply $\omega_c = \frac{1}{4}$. Stability reserve of closed system $\Delta\varphi$:

$$\Delta\varphi = -\pi - \varphi_c = -\pi - \mathrm{arc}G_h(j\omega_c) = \arctan(\omega_c T_2) - \arctan(\omega_c T_B)$$

$$\Delta\varphi = \arctan\left(\frac{1}{2}\right) - \arctan(2)$$



Chọn $a = 4$

Exercise 2

a) **(1 marks)** we denote

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix},\ B = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix},\ \underline{c} = \begin{pmatrix} a \\ 0 \\ 1 \end{pmatrix} \Rightarrow \begin{cases} \dfrac{d\underline{x}}{dt} = A\underline{x} + B\underline{u} \\ y = \underline{c}^T \underline{x} \end{cases}$$

i. **(0.5 marks)**
The equivalent polynomial of matrix A will be:
$\det(sI - A) = (s-1)(s^2 - 3s + 1)$. This system is not stable;

ii. **(0.5 marks)** $\mathrm{Rank}(B, AB, A^2B) = 3 \Rightarrow$ This system is controllable

b) (1 marks)

i. **(0.5 marks)** $N = \begin{pmatrix} \underline{c}^T \\ \underline{c}^T A \\ \underline{c}^T A^2 \end{pmatrix} = \begin{pmatrix} a & 0 & 1 \\ a & 1 & a+1 \\ a & a+3 & 2a+2 \end{pmatrix}$

ii. **(0.5 marks)** $\det(N) = -a(a^2 + a - 1)$

In order to obtain the observability property: $\det(N) \neq 0 \Leftrightarrow a \neq 0; \frac{-1 \pm \sqrt{5}}{2}$

c) (2 marks)

i. **(1 marks)** State Feedback control is $\underline{u} = \underline{\omega} - R\underline{x}$ với $R = (r_1 \quad r_2 \quad r_3)$ sastisfy eigen values of $(A - \underline{b}\underline{r}^T)$ inside $(-2, 0)$ and we select all of eigen values being -1), we obtain based on Ackermann
$[r_1 \quad r_2 \quad r_3] = [0 \quad 0 \quad 1] M^{-1} \Phi_R(A) = [-4, -24, 11]$

ii. **(1 marks)** Observer

i. **(0.5 marks)** $\hat{\underline{x}}$ is the root of difference equation
$$\frac{d\hat{\underline{x}}}{dt} = A\hat{\underline{x}} + \underline{b}u + L\left(y - \underline{c}^T \hat{\underline{x}}\right).$$

ii. **(0.5 marks)** Find matrix L to obtain that $(A - L\underline{c}^T)$ have all eigenvalues being -3 (faster than e^{-2t}). Using Ackermann:

$$L = \Phi_L(A)N^{-1}\begin{bmatrix}0\\0\\1\end{bmatrix} = [28, 77, -15]^T$$

(1đ) **Drawing. The closed system is not controllable because $\hat{\underline{x}}$ converge to $\underline{x}$**

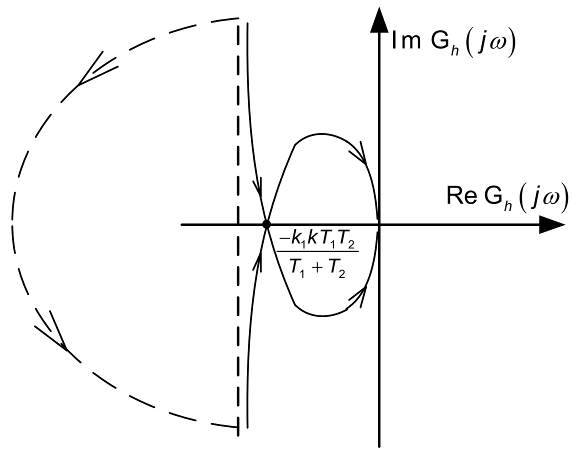
a) **(0.5 đ)** Selecting the state feedback controller $\underline{u} = \underline{\omega} - R\underline{x}$ with $R = \begin{bmatrix}-4 & -24 & 11\\0 & 0 & 0\end{bmatrix}$

SOLUTION OF EXAM 02
Subject: CONTROL THEORY

Exercise 1 **(5 marks)**:

a) (1.5 marks)

i. **(1.5 đ) We have**:



(0.5đ) $0 < k_1 < 0.2$

i. **(0.5 đ)**

Because of stable closed loop system, there exists the Limitation

$$\lim_{t\to\infty}\big(u(t)-y(t)\big)=\lim_{s\to 0} s\big(U(s)-Y(s)\big)=\lim_{s\to 0} sU(s)\left(1-\frac{k_2k_1G(s)}{1+k_1G(s)}\right);$$

Because $u(t)=1(t)$, we imply $\lim_{s\to 0} G(s)=+\infty$ and

$\lim_{t\to\infty}\big(u(t)-y(t)\big)=1-k_2$, we obtain $k_2=1$;

b) (2 marks)

i. **(1 marks)** $G(s)=\dfrac{k}{s(1+T_2s)^2}$

($a=4$): $T_I=T_1+4T_2,\ k_p=\dfrac{T_I}{8kT_2^2},\ T_D=\dfrac{4T_1T_2}{T_I},\ T=4T_2$ where

$k=10,\ T_1=T_2=1$ we obtain $T_I=5,\ k_p=\dfrac{1}{16},\ T_D=0.8,\ T=4$

ii. **(1 mark)** Stability reserve of closed system does not depend on $R_2(s)$. We need to obtain the open system with transfer function:

$$G_h(s)=R_1(s)G(s)=k_p\left(1+\frac{1}{T_Is}+T_Ds\right)\cdot\frac{k}{s(1+T_1s)(1+T_2s)}$$

$$=\frac{k_p(1+T_As)(1+T_Bs)}{T_Is}\cdot\frac{k}{s(1+T_1s)(1+T_2s)}=\frac{k_pk(1+T_Bs)}{T_Is^2(1+T_2s)}$$

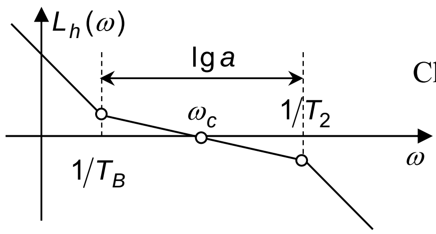
We select $T_A=T_1$ with $T_A+T_B=T_I$, $T_AT_B=T_IT_D$, $T_B=4T_2>T_2$, we obtain the Bode diagram of open system as follows.

$\omega_c = \dfrac{1}{\sqrt{T_B T_2}}$.

$T_B = 8,\ T_1 = T_2 = 2$ we imply $\omega_c = \dfrac{1}{4}$. Stability reserve of closed system $\Delta\varphi$:

$\Delta\varphi = -\pi - \varphi_c = -\pi - \mathrm{arc}G_h(j\omega_c) = \arctan(\omega_c T_2) - \arctan(\omega_c T_B)$

$\Delta\varphi = \arctan\left(\dfrac{1}{2}\right) - \arctan(2)$



Chọn $a = 4$

Exercise 2

a) **(1 marks)** we denote

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix},\ B = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix},\ \underline{c} = \begin{pmatrix} a \\ 0 \\ 1 \end{pmatrix} \Rightarrow \begin{cases} \dfrac{d\underline{x}}{dt} = A\underline{x} + B\underline{u} \\ y = \underline{c}^T \underline{x} \end{cases}$$

i. (0.5 marks)

The equivalent polynomial of matrix A will be:

$\det(sI - A) = (s-1)(s^2 - 3s + 1)$. This system is not stable;

ii. **(0.5 marks)** $\mathrm{Rank}(B, AB, A^2B) = 3 \Rightarrow$ This system is controllable

b) (1 marks)

i. **(0.5 marks)** $N = \begin{pmatrix} \underline{c}^T \\ \underline{c}^T A \\ \underline{c}^T A^2 \end{pmatrix} = \begin{pmatrix} a & 0 & 1 \\ a & 1 & a+1 \\ a & a+3 & 2a+2 \end{pmatrix}$

ii. **(0.5 marks)** $\det(N) = -a(a^2 + a - 1)$

In order to obtain the observability property: $\det(N) \neq 0 \Leftrightarrow a \neq 0; \dfrac{-1 \pm \sqrt{5}}{2}$

c) (2 marks)

i. **(1 marks)** State Feedback control is $\underline{u} = \underline{\omega} - R\underline{x}$ với $R = (r_1 \quad r_2 \quad r_3)$ sastisfy eigen values of $\left(A - \underline{b}\underline{r}^T\right)$ inside $(-2, 0)$ and we select all of eigen values being -1), we obtain based on Ackermann

$\left[r_1 \quad r_2 \quad r_3\right] = \left[0 \quad 0 \quad 1\right] M^{-1} \Phi_R(A) = \left[-4, -24, 11\right]$

ii. **(1 marks)** Observer

i. **(0.5 marks)** $\underline{\hat{x}}$ is the root of difference equation

$\dfrac{d\underline{\hat{x}}}{dt} = A\underline{\hat{x}} + \underline{b}u + L\left(y - \underline{c}^T \underline{\hat{x}}\right)$.

ii. **(0.5 marks)** Find matrix L to obtain that $\left(A - L\underline{c}^T\right)$ have all eigenvalues being -3 (faster than e^{-2t}). Using Ackermann:

$$L = \Phi_L(A)N^{-1}\begin{bmatrix}0\\0\\1\end{bmatrix} = [28, 77, -15]^T$$

(1đ) Drawing. The closed system is not controllable because $\hat{\underline{x}}$ converge to $\underline{x}$

(0.5 đ) Selecting the state feedback controller $\underline{u} = \underline{\omega} - R\underline{x}$ with $R = \begin{bmatrix}-9 & -16 & 17\\0 & 0 & 0\end{bmatrix}$