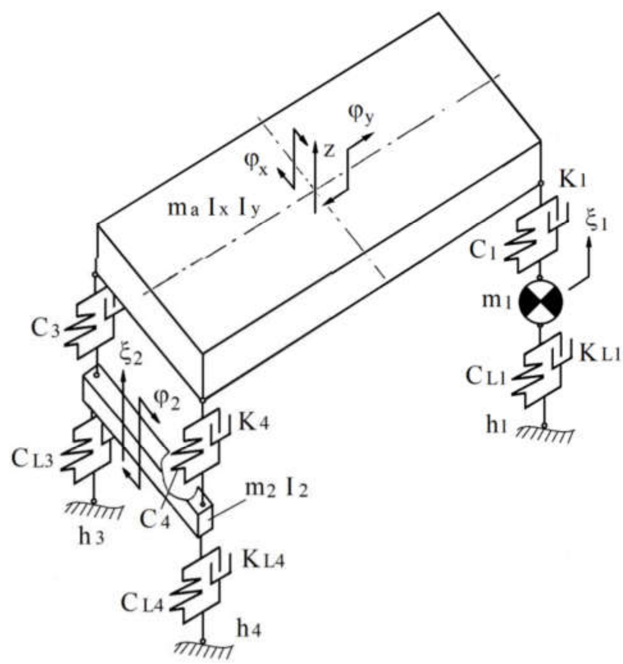
Hanoi University of Science and Technology **School of Transportation Engineering** **Exam No. 1 No. of pages: 1**		**FINAL EXAM** **TE4240: Vehicle dynamics** *Date: 14.06.2018* *Duration: 90 min* ***(Open printed slide and textbook,*** ***Exam Papers and Problem sheet must be handed in)***
Sign	Subject group leader: Dr. Trinh Minh Hoang	D.Head of the Department: Dr. Dam Hoang Phuc

Problem

Given a mechanical model of truck with the suspensions and wheels in the figure below. In the model, assuming that the sprung mass is described as a plate with properties mass m_a, I_x, I_y. The sprung mass has 3 degrees of freedoms (DOF), which are vertical displacement z, angular φ_x (around longitudinal axis x) and angular φ_y (around yaw axis y). The front unsprungs masses are described as two mass (m_{11}, m_{12}) with two displacements ξ_{11} and ξ_{12} along vertical axis. The rear unsprung mass is described as a bar with two DOF: vertical displacement ξ_2 and angular φ_2 around the longitudinal axis x. Wheels contact as points to the road surface, in the general case here may occur the phenomenon of separation of wheels from the road surface. The velocity of the car is constant.

Requirements: Develop a system of differential equations describing vertical dynamics of the vehicle.



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VIỆN CƠ KHI ĐỘNG LỰC

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Học phần: Động lực học ô tô Mã HP: TE 4240

Mã lớp học: 102693 Mã lớp thi: 83942

Bài thi: [ ] giữa kỳ [✓] cuối kỳ 20172 Năm học: 2017 - 2018 Ngày thi: 14/6/2018

Điểm của bài thi	Chữ ký của (các) cán bộ chấm thi	Chữ ký của cán bộ coi thi
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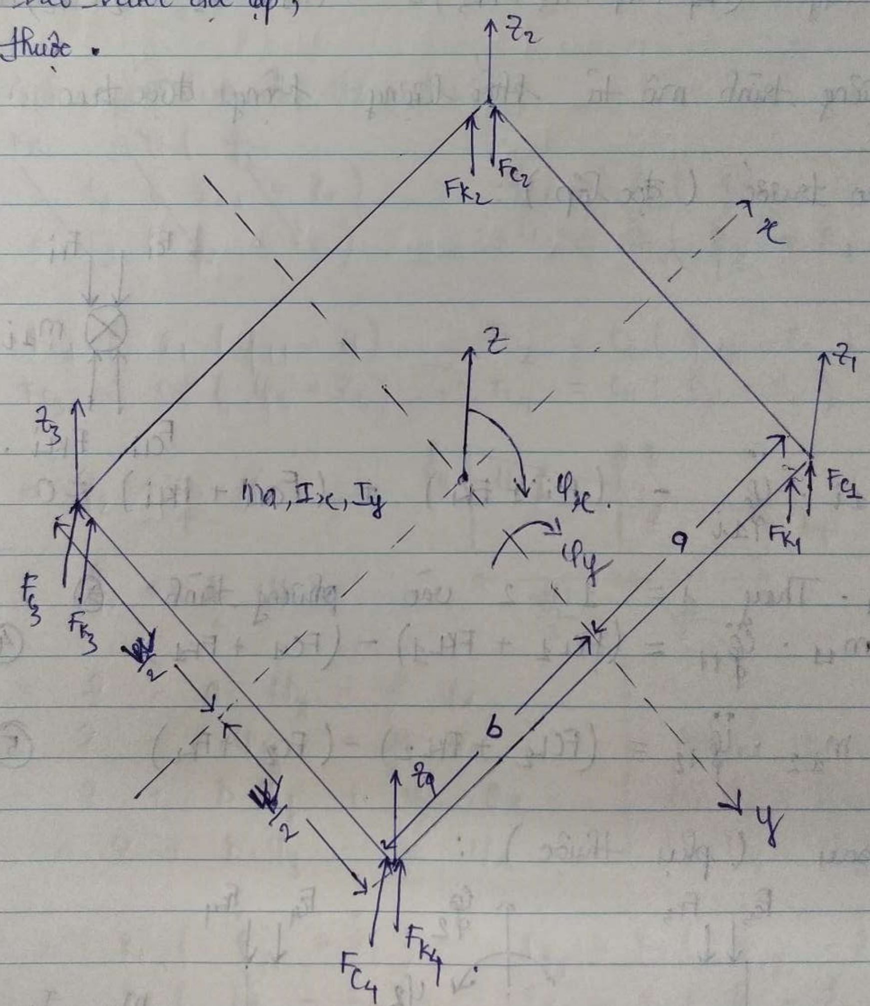
- Hệ thống treo trước độc lập, sau phụ thuộc.



⊕ Phương trình với khối lượng được treo:

- Theo phương z:

$$-m_h\ddot{z} + F_{C1} + F_{K1} + F_{C2} + F_{K2} + F_{C3} + F_{K3} + F_{C4} + F_{K4} = 0$$

(1) $\Leftrightarrow m_h\ddot{z} = F_{C1} + F_{K1} + F_{C2} + F_{K2} + F_{C3} + F_{K3} + F_{C4} + F_{K4}$ (1)

- Theo góc quay φ_x:

$$-I_x \cdot \ddot{\varphi}_x + (F_{K2} + F_{C2} + F_{C3} + F_{K3}) \cdot \frac{W}{2} - (F_{K1} + F_{C1} + F_{C4} + F_{K4})\frac{W}{2} = 0$$

(1) $\Leftrightarrow I_x \cdot \ddot{\varphi}_x = (F_{K2} + F_{C2} + F_{C3} + F_{K3})\frac{W}{2} - (F_{K1} + F_{C1} + F_{C4} + F_{K4})\frac{W}{2}$ (2)

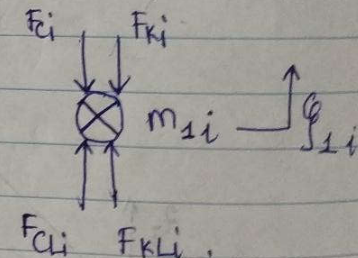
- Theo góc quay φ_y:

$$-I_y \cdot \ddot{\varphi}_y + (F_{K4} + F_{C4} + F_{C3} + F_{K3})b - (F_{C1} + F_{K1} + F_{C2} + F_{K2})a = 0$$

(1) $\Leftrightarrow I_y \cdot \ddot{\varphi}_y = (F_{K4} + F_{C4} + F_{C3} + F_{K3})b - (F_{C1} + F_{K1} + F_{C2} + F_{K2})a$ (3)

⊕ Phương trình mô tả khối lượng không được treo.

- Treo trước (độc lập):

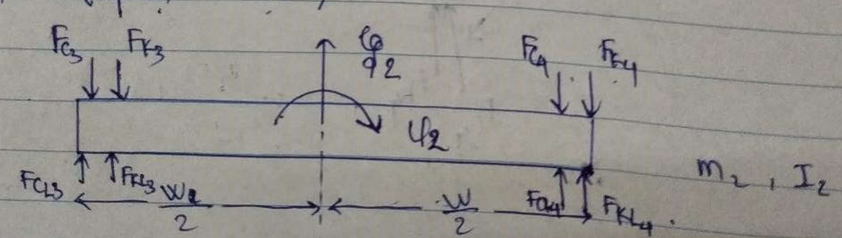


$$-m_{1i} \cdot \ddot{q}_{1i} - (F_{Ci} + F_{Ki}) + (F_{CLi} + F_{KLi}) = 0 \quad (i = 1; 2).$$ (*)

$\Rightarrow$ Thay $i = 1; 2$ vào phương trình (*):

(2) $\Rightarrow \begin{cases} m_{11} \cdot \ddot{q}_{11} = (F_{CL1} + F_{KL1}) - (F_{C1} + F_{K1}) & (4) \\ m_{12} \cdot \ddot{q}_{12} = (F_{CL2} + F_{KL2}) - (F_{C2} + F_{K2}) & (5) \end{cases}$

- Treo sau (phụ thuộc):



$$- m_2 \cdot \ddot{\xi}_2 - (F_{C3} + F_{K3} + F_{C4} + F_{K4}) + (F_{CL3} + F_{KL3} + F_{CL4} + F_{KL4}) = 0$$

(1) $\Leftrightarrow m_2 \cdot \ddot{\xi}_2 = -(F_{C3} + F_{K3} + F_{C4} + F_{K4}) + (F_{CL3} + F_{KL3} + F_{CL4} + F_{KL4})$ (6).

- Theo góc quay φ_2:

$$- I_2 \cdot \ddot{\varphi}_2 + \frac{W}{2}(F_{C4} + F_{K4} + F_{CL3} + F_{KL3}) - \frac{W}{2}(F_{C3} + F_{K3} + F_{CL4} + F_{KL4}) = 0$$

(1) $\Leftrightarrow I_2 \cdot \ddot{\varphi}_2 = \frac{W}{2}(F_{C4} + F_{K4} + F_{CL3} + F_{KL3}) - \frac{W}{2}(F_{C3} + F_{K3} + F_{CL4} + F_{KL4})$ (7)

- Hệ phương trình (1); (2); (3); (4); (5); (6); (7) mô tả động lực học theo phương thẳng đứng của xe.

- Xác định các lực hệ thống treo:

~~$F_{Ci} = C_i(\xi_i$~~

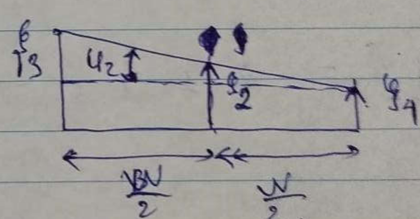
~~$F_{C1} = C_1(\xi_{11} - z_1)$; $F_{C2} = C_2(\xi_{12} - z_2)$~~

~~$F_{C3} = C_3(\xi_2 - z_3)$; $F_{C4} = C_4(\xi_2$ [illegible] $z_4)$~~

(4) Với $F_{C1} = c_1(\xi_{11} - z_1)$; $F_{C2} = c_2(\xi_{12} - z_2)$

$F_{C3} = c_3(\xi_3 - z_3)$; $F_{C4} = c_4(\xi_4 - z_4)$.

Với: $\begin{cases} \xi_3 = \xi_2 + \frac{B}{2} \cdot \varphi_2 \\ \xi_4 = \xi_2 - \frac{B}{2} \varphi_2 \end{cases}$



$$\begin{cases} z_1 = z - a \cdot \varphi_y - \frac{W}{2} \varphi_x \\ z_2 = z - a \cdot \varphi_y + \frac{W}{2} \varphi_x \\ z_3 = z + b \cdot \varphi_y + \frac{W}{2} \cdot \varphi_x \\ z_4 = z + b \cdot \varphi_y - \frac{W}{2} \varphi_x \end{cases}$$

Giả sử φ_x, φ_y nhỏ.
$\tan \varphi_x \approx \varphi_x$
$\tan \varphi_y \approx \varphi_y$.

- $F_{K1} = K_1(\dot{\xi}_{11} - \dot{z}_1)$; $F_{K2} = K_2(\dot{\xi}_{12} - \dot{z}_2)$

$F_{K3} = K_3(\dot{\xi}_3 - \dot{z}_3)$; $F_{K4} = K_4(\dot{\xi}_4 - \dot{z}_4)$.

* Xác định các phản lực của lốp :

- Khi không tách bánh :

$$F_{Ci} + F_{Ki} = C_{Li}(h_i - q_i) + K_{Li}(\dot{h}_i - \dot{q}_i)$$

với bánh xe ~~trước~~ thứ i ; $i = 1 \div 4$.

(1) Đặt $q_1 = q_{11}$; $q_2 = q_{12}$.

— lâu ký hiệu

- Khi tách bánh xe :

$$F_{Ci} + F_{Ki} = \begin{cases} C_{Li}(h_i - q_i) + K_{Li}(\dot{h}_i - \dot{q}_i) & ; \text{khi } h_i - (q_i - f_{ti}) \geq 0 \\ -F_{Gi} & ; \text{khi } h_i - (q_i - f_{ti}) < 0 \end{cases}$$

trong đó : $f_{ti} = \frac{F_{Gi}}{C_{Li}}$.

$$F_{G1;2} = g\left(m_{11;12} + \frac{m_a \cdot b}{2L}\right)$$

$$F_{G3} = F_{G4} = g\left(\frac{m_2}{2} + \frac{m_a \cdot a}{2L}\right) .$$

~~Đặt~~ : $q_1 = q_{11}$; $q_2 = q_{12}$;

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Mã lớp học: 102693 Mã lớp thi: 83942

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Điểm của bài thi	Chữ ký của (các) cán bộ chấm thi	Chữ ký của cán bộ coi thi
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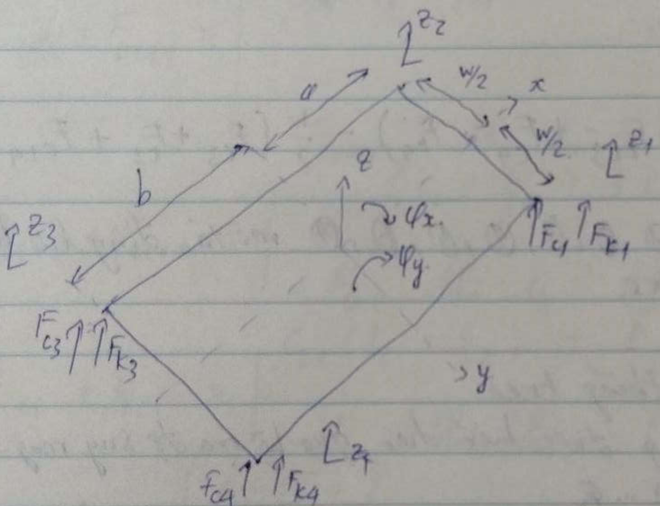


+ Theo phương z:

(1) $-m_a \cdot \ddot{z} + (F_{C1} + F_{K1}) + (F_{C2} + F_{K2}) + (F_{C3} + F_{K3}) + (F_{C4} + F_{K4}) = 0$ (1)

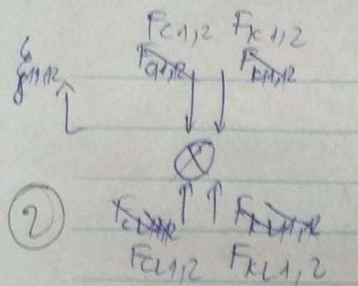
+ Theo góc quay φ_x:

(1) $-I_x \cdot \ddot{\varphi}_x + \frac{w}{2}(F_{C2} + F_{K2} + F_{C3} + F_{K3}) - \frac{w}{2}(F_{C1} + F_{K1} + F_{C4} + F_{K4}) = 0$ (2)

+ Theo góc quay φ_y:

(1) $-I_y \ddot{\varphi}_y - a(F_{C1} + F_{K1} + F_{C2} + F_{K2}) + b(F_{C3} + F_{K3} + F_{C4} + F_{K4}) = 0$ (3)

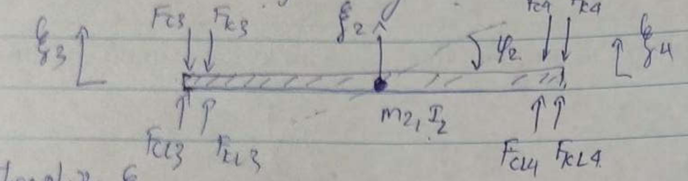
+ Vì các khối lượng không được treo ở cầu trước:



$$-m_{11}\cdot\ddot{\xi}_{11}-(F_{C11}+F_{K11})+(F_{CL11}+F_{KL11})=0 \quad (4)$$

$$-m_{12}\cdot\ddot{\xi}_{12}-(F_{C2}+F_{K2})+(F_{CL2}+F_{KL2})=0 \quad (5)$$

+ Với các khối lượng không được treo ở cầu sau:



Theo phương ξ_2:

$$-m_2\cdot\ddot{\xi}_2-(F_{C3}+F_{K3}+F_{C4}+F_{K4})+(F_{CL3}+F_{KL3}+F_{CL4}+F_{KL4})=0 \quad (6)$$

Theo góc quay φ_2:

$$-I_2\cdot\ddot{\varphi}_2+\frac{W}{2}(F_{CL3}+F_{KL3}+F_{C4}+F_{K4})-\frac{W}{2}(F_{C3}+F_{K3}+F_{CL4}+F_{KL4})=0 \quad (7)$$

Các phương trình vi phân (1), (2), (3), (4), (5), (6), (7) mô tả động lực học theo phương thẳng đứng của xe.

× Xác định các lực hệ thống treo.

- Các chuyển vị thẳng đứng được biểu diễn qua hệ tọa độ suy rộng:

$$z_1 = z - a\cdot\varphi_y - \frac{W}{2}\cdot\varphi_x$$
$$z_2 = z - a\cdot\varphi_y + \frac{W}{2}\cdot\varphi_x$$
$$z_3 = z + b\cdot\varphi_y + \frac{W}{2}\cdot\varphi_x$$
$$z_4 = z + b\cdot\varphi_y - \frac{W}{2}\cdot\varphi_x$$

$$\xi_3 = \xi_2 + \frac{W}{2}\cdot\varphi_2$$

$F_{C3}, F_{C4}, F_{K3}, F_{K4}$?

$$\xi_4 = \xi_2 - \frac{W}{2}\cdot\varphi_2$$

- Các lực:

$$F_{C1} = C_1(\xi_{11} - z_1) \qquad F_{C2} = C_2(\xi_{12} - z_2)$$

$$F_{K1} = K_1(\dot{\xi}_{11} - \dot{z}_1) \qquad F_{K2} = K_2(\dot{\xi}_{12} - \dot{z}_2)$$

* Xác định phản lực lốp:

- Không tách bánh xe:

$$F_{CL1} + F_{KL1} = C_{L1}(h_1 - \xi_{11}) + K_{L1}(\dot{h}_1 - \dot{\xi}_{11})$$

$$F_{CL2} + F_{KL2} = C_{L2}(h_2 - \xi_{12}) + K_{L2}(\dot{h}_2 - \dot{\xi}_{12})$$

$$F_{CL3} + F_{KL3} = C_{L3}(h_3 - \xi_3) + K_{L3}(\dot{h}_3 - \dot{\xi}_3)$$

$$F_{CL4} + F_{KL4} = C_{L4}(h_4 - \xi_4) + K_{L4}(\dot{h}_4 - \dot{\xi}_4).$$

- Tách bánh xe.

$$F_{CL11} + F_{KL11} = \begin{cases} C_{L1}(h_{1} - \xi_{11}) + K_{L1}(\dot{h}_1 - \dot{\xi}_{11}) & \text{khi } h_1 - (\xi_{11} - f_{t1}) \geq 0 \\ -F_{G1} & \text{khi } h_1 - (\xi_{11} - f_{t1}) < 0 \end{cases}$$

(2)

$$F_{CL12} + F_{KL12} = \begin{cases} C_{L2}(h_2 - \xi_{12}) + K_{L2}(\dot{h}_2 - \dot{\xi}_{12}) & \text{khi } h_2 - (\xi_{12} - f_{t2}) \geq 0 \\ -F_{G2} & \text{khi } h_2 - (\xi_{12} - f_{t2}) < 0 \end{cases}$$

$$F_{CLi} + F_{KLi} = \begin{cases} C_{Li}(h_i - \xi_i) + K_{Li}(\dot{h}_i - \dot{\xi}_i) & \text{khi } h_i - (\xi_i - f_{ti}) \geq 0 \\ -F_{Gi} & \text{khi } h_i - (\xi_i - f_{ti}) < 0 \end{cases}$$

$(i = \overline{3;4})$.

Trong đó: $f_{ti} = \dfrac{F_{Gi}}{C_{Li}}$ $(i = \overline{1;4})$.

$L = a + b$

$$F_{G1} = g\left(m_{11} + \frac{m_a \cdot b}{2L}\right)$$

$$F_{G2} = g\left(m_{12} + \frac{m_a \cdot b}{2L}\right)$$

$$F_{G3} = F_{G4} = g\left(\frac{m_2}{2} + \frac{m_a \cdot a}{2L}\right)$$

SCHOOL OF TRANSPORTATION ENGINEERING

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Subject: Vehicle dynamics   Subject code: TE4240

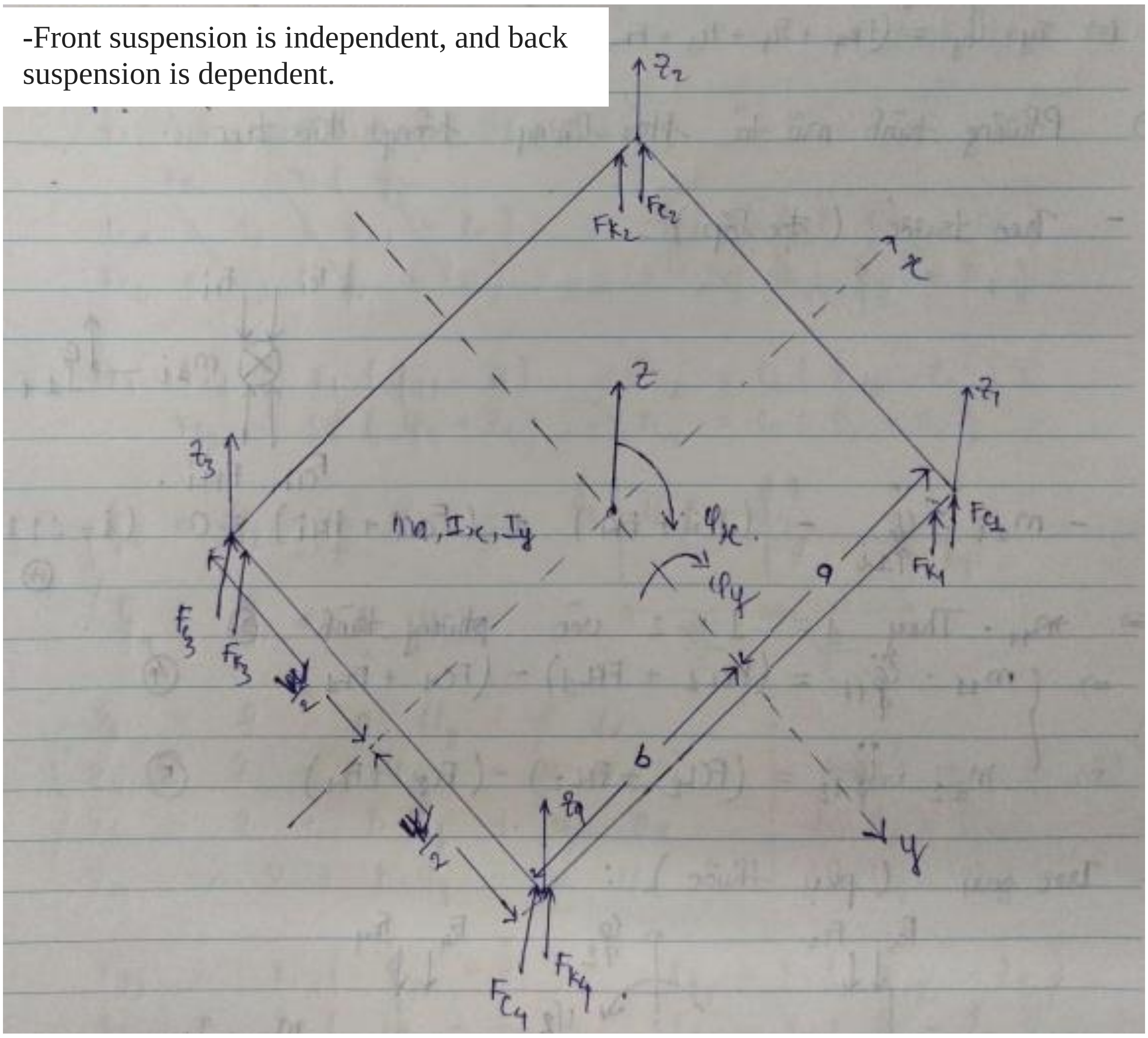
Class code : 102693   Exam code : 83942

Exam type : Middle ☑ Final   School year: 2017-2018   Exam date: 14/06/2018

Exam score	Signature of examiners	Signature of exam monitor
9	(Signed)	(Signed)

Sheet…/…

-Front suspension is independent, and back suspension is dependent.



+ Equations with mass are suspended

- For axis z:

$$-m_n\ddot{z} + F_{C1} + F_{K1} + F_{C2} + F_{K2} + F_{C3} + F_{K3} + F_{C4} + F_{K4} = 0$$

$$\Leftrightarrow m_n\ddot{z} = F_{C1} + F_{K1} + F_{C2} + F_{K2} + F_{C3} + F_{K3} + F_{C4} + F_{K4} \quad (1)$$

- For rotating angle φ_x

$$-I_x \cdot \ddot{\varphi}_x + (F_{K2} + F_{C2} + F_{C3} + F_{K3}) \cdot \frac{W}{2} - (F_{K1} + F_{C1} + F_{C4} + F_{K4})\frac{W}{2}$$

$$\Leftrightarrow I_x \cdot \ddot{\varphi}_x = (F_{K2} + F_{C2} + F_{C3} + F_{K3})\frac{W}{2} - (F_{K1} + F_{C1} + F_{C4} + F_{K4})\frac{W}{2} \quad (2)$$

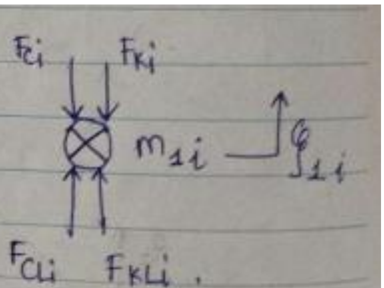
- For rotating angle φ_y

$$-I_y \cdot \ddot{\varphi}_y + (F_{K4} + F_{C4} + F_{C3} + F_{K3})b - (F_{C1} + F_{K1} + F_{C2} + F_{K2})a = 0$$

$$\Leftrightarrow I_y \cdot \ddot{\varphi}_y = (F_{K4} + F_{C4} + F_{C3} + F_{K3})b - (F_{C1} + F_{K1} + F_{C2} + F_{K2})a \quad (3)$$

+ Equations of the unsuspended mass

- Front suspension (independent):

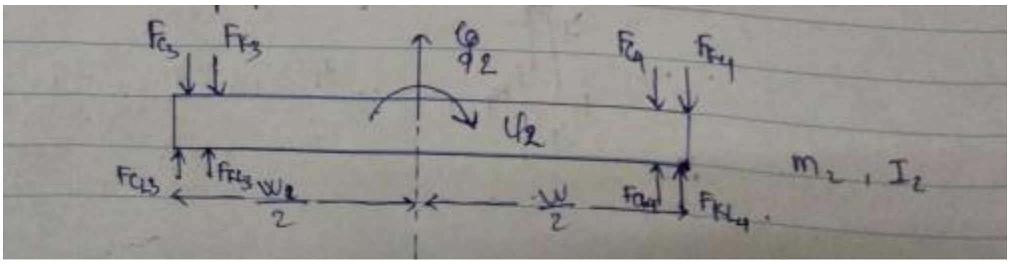


$$-m_{1i} \cdot \ddot{\varphi}_{1i} - (F_{Ci} + F_{Ki}) + (F_{CLi} + F_{KLi}) = 0 \quad (i = 1;2) \quad (*)$$

Substituting i =1,2 into the equation (*):

$$\Rightarrow \begin{cases} m_{11} \cdot \ddot{\varphi}_{11} = (F_{CL1} + F_{KL1}) - (F_{C1} + F_{K1}) & (4) \\ m_{12} \cdot \ddot{\varphi}_{12} = (F_{CL2} + F_{KL2}) - (F_{C2} + F_{K2}) & (5) \end{cases}$$

- Back suspension (dependent):



$$-m_2 \cdot \ddot{\xi}_2 - (F_{C3}+F_{K3}+F_{C4}+F_{K4}) + (F_{CL3}+F_{KL3}+F_{CL4}+F_{KL4}) = 0$$

$$\Leftrightarrow m_2 \cdot \ddot{\xi}_2 = -(F_{C3}+F_{K3}+F_{C4}+F_{K4}) + (F_{CL3}+F_{KL3}+F_{CL4}+F_{KL4}) \quad (6)$$

- For rotating angle φ_2

$$-I_2 \cdot \ddot{\varphi}_2 + \frac{W}{2}(F_{C4}+F_{K4}+F_{CL3}+F_{KL3}) - \frac{W}{2}(F_{C3}+F_{K3}+F_{CL4}+F_{KL4}) = 0$$

$$\Leftrightarrow I_2 \cdot \ddot{\varphi}_2 = \frac{W}{2}(F_{C4}+F_{K4}+F_{CL3}+F_{KL3}) - \frac{W}{2}(F_{C3}+F_{K3}+F_{CL4}+F_{KL4}) \quad (7)$$

- Equations 1, 2, 3, 4, 5, 6,7 describe dynamics according to vertical direction of the truck

- Determine the forces of the suspension:

~~$F_{Ci} = C_i(\xi_i$~~

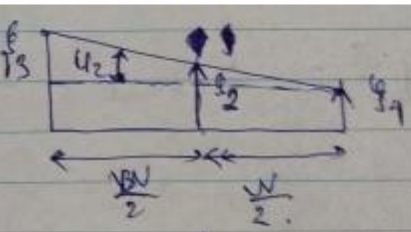
~~$F_{C1} = C_1(\xi_{11} - z_1)$; $F_{C2} = C_2(\xi_{12} - z_2)$~~

~~$F_{C3} = C_3(\xi_2 - z_3)$; $F_{C4} = C_4(\xi_2 - z_4)$~~

$$F_{C1} = C_1(\xi_{11} - z_1) \; ; \; F_{C2} = C_2(\xi_{12} - z_2)$$

$$F_{C3} = C_3(\xi_3 - z_3) \; ; \; F_{C4} = C_4(\xi_4 - z_4)$$

with
$$\begin{cases} \xi_3 = \xi_2 + \frac{B}{2}\cdot\varphi_2 \\ \xi_4 = \xi_2 - \frac{B}{2}\varphi_2 \end{cases}$$



$$\begin{cases} z_1 = z - a\cdot\varphi_y - \frac{W}{2}\varphi_x \\ z_2 = z - a\cdot\varphi_y + \frac{W}{2}\varphi_x \\ z_3 = z + b\cdot\varphi_y + \frac{W}{2}\cdot\varphi_x \\ z_4 = z + b\cdot\varphi_y - \frac{W}{2}\varphi_x \end{cases}$$

Assuming φ_x , φ_y are small

$\tan\varphi_x \approx \varphi_x$

$\tan\varphi_y \approx \varphi_y$

- $F_{K1} = K_1(\dot{\xi}_{11} - \dot{z}_1)$ $\quad F_{K2} = K_2(\dot{\xi}_{12} - \dot{z}_2)$

$F_{K3} = K_3(\dot{\xi}_3 - \dot{z}_3)$ $\quad F_{K4} = K_4(\dot{\xi}_4 - \dot{z}_4)$.

* Determine the force of the tire

- when not separating the wheel:

$$F_{Ci} + F_{Ki} = C_{Li}(h_i - \xi_i) + K_{Li}(\dot{h}_i - \dot{\xi}_i)$$

with the wheel i: ; $i = 1 \div 4$.

Put: $\xi_1 = \xi_{11}$; $\xi_2 = \xi_{12}$.

- when separating the wheel:

$$F_{Ci} + F_{Ki} = \begin{cases} C_{Li}(h_i - \xi_i) + K_{Li}(\dot{h}_i - \dot{\xi}_i) & \text{; khi } h_i - (\xi_i - f_{ti}) \geq 0 \\ -F_{Gi} & \text{; khi } h_i - (\xi_i - f_{ti}) < 0 \end{cases}$$

while : $f_{ti} = \dfrac{F_{Gi}}{C_{Li}}$.

$$F_{G1;2} = g\left(m_{11;12} + \frac{m_a \cdot b}{2L}\right)$$

$$F_{G3} = F_{G4} = g\left(\frac{m_2}{2} + \frac{m_a \cdot a}{2L}\right).$$

Put: $\xi_1 = \xi_{11}$; $\xi_2 = \xi_{12}$;

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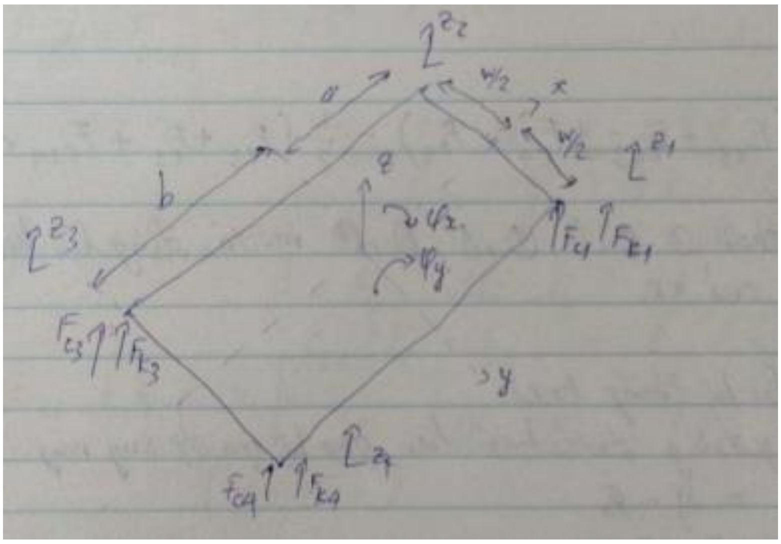


+ For axis z:

$$-m_q \cdot \ddot{z} + (F_{c1}+F_{k1}) + (F_{c2}+F_{k2}) + (F_{c3}+F_{k3}) + (F_{c4}+F_{k4}) = 0 \quad (1)$$

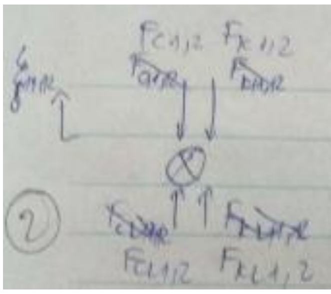
+ For rotating angle φ_x

$$-I_x \cdot \ddot{\varphi}_x + \frac{w}{2}(F_{c2}+F_{k2}+F_{c3}+F_{k3}) - \frac{w}{2}(F_{c1}+F_{k1}+F_{c4}+F_{k4}) = 0 \quad (2)$$

+ For rotating angle φ_y

$$-I_y \ddot{\varphi}_y - a(F_{c1}+F_{k1}+F_{c2}+F_{k2}) + b(F_{c3}+F_{k3}+F_{c4}+F_{k4}) = 0 \quad (3)$$

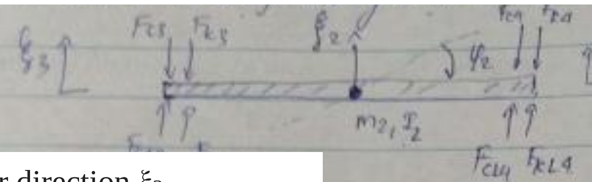
+ with masses not hanging in the front bridge



$$-m_{11}\cdot\ddot{\xi}_{11} - (F_{C1} + F_{K1}) + (F_{CL1} + F_{KL1}) = 0 \quad (3)$$

$$-m_{12}\cdot\ddot{\xi}_{12} - (F_{C2} + F_{K2}) + (F_{CL2} + F_{KL2}) = 0 \quad (4)$$

+ with masses are hanged in the back bridge



For direction ξ_2

$$-m_2\cdot\ddot{\xi}_2 - (F_{C3} + F_{K3} + F_{C4} + F_{K4}) + (F_{CL3} + F_{KL3} + F_{CL4} + F_{KL4}) = 0 \quad (6)$$

For rotating angle φ_2

$$-I_2\cdot\ddot{\varphi}_2 + \frac{W}{2}(F_{CL3} + F_{KL3} + F_{C4} + F_{K4}) - \frac{W}{2}(F_{C3} + F_{K3} + F_{CL4} + F_{KL4}) = 0 \quad (7)$$

Equations 1, 2, 3, 4, 5, 6,7 describe dynamics according to vertical direction of the truck

* Determine the forces of the suspension:

Vertical displacements are represented by generalized coordinates

$$z_1 = z - a\cdot\varphi_y - \frac{W}{2}\cdot\varphi_x$$
$$z_2 = z - a\cdot\varphi_y + \frac{W}{2}\cdot\varphi_x$$
$$z_3 = z + b\cdot\varphi_y + \frac{W}{2}\cdot\varphi_x$$
$$z_4 = z + b\cdot\varphi_y - \frac{W}{2}\cdot\varphi_x$$

$$\xi_3 = \xi_2 + \frac{W}{2}\cdot\varphi_2$$

$$\xi_4 = \xi_2 - \frac{W}{2}\cdot\varphi_2$$

- Forces:

$$F_{C1} = C_1(\xi_{11} - z_1) \qquad F_{C2} = C_2(\xi_{12} - z_2)$$

$$F_{K1} = k_1(\dot{\xi}_{11} - \dot{z}_1) \qquad F_{K2} = k_2(\dot{\xi}_{12} - \dot{z}_2)$$

* Determine tire reaction

- when not separating the wheel:

$$F_{CL1} + F_{KL1} = C_{L1}(h_1 - \xi_{11}) + k_{L1}(\dot{h}_1 - \dot{\xi}_{11})$$

$$F_{CL2} + F_{KL2} = C_{L2}(h_2 - \xi_{12}) + k_{L2}(\dot{h}_2 - \dot{\xi}_{12})$$

$$F_{CL3} + F_{KL3} = C_{L3}(h_3 - \xi_3) + k_{L3}(\dot{h}_3 - \dot{\xi}_3)$$

$$F_{CL4} + F_{KL4} = C_{L4}(h_4 - \xi_4) + k_{L4}(\dot{h}_4 - \dot{\xi}_4).$$

- when separating the wheel:

$$F_{CL1} + F_{KL1} = \begin{cases} C_{L1}(h_1 - \xi_{11}) + k_{L1}(\dot{h}_1 - \dot{\xi}_{11}) & \text{if } h_1 - (\xi_{11} - f_{t1}) \geq 0 \\ -F_{G1} & \text{if } h_1 - (\xi_{11} - f_{t1}) < 0 \end{cases}$$

$$F_{CL2} + F_{KL2} = \begin{cases} C_{L2}(h_2 - \xi_{12}) + k_{L2}(\dot{h}_2 - \dot{\xi}_{12}) & \text{if } h_2 - (\xi_{12} - f_{t2}) \geq 0 \\ -F_{G2} & \text{if } h_2 - (\xi_{12} - f_{t2}) < 0 \end{cases}$$

$$F_{CLi} + F_{KLi} = \begin{cases} C_{Li}(h_i - \xi_i) + k_{Li}(\dot{h}_i - \dot{\xi}_i) & \text{if } h_i - (\xi_i - f_{ti}) \geq 0 \\ -F_{Gi} & \text{if } h_i - (\xi_i - f_{ti}) < 0 \end{cases}$$

$(i = \overline{3;4})$.

While: $f_{ti} = \dfrac{F_{Gi}}{C_{Li}}$ $(i = \overline{1;4})$.

$$L = a + b$$

$$F_{G1} = g\left(m_{11} + \frac{m_a \cdot b}{2L}\right)$$

$$F_{G2} = g\left(m_{12} + \frac{m_a \cdot b}{2L}\right)$$

$$F_{G3} = F_{G4} = g\left(\frac{m_2}{2} + \frac{m_a \cdot a}{2L}\right)$$